**Fun event next week**

Justine Cassell
“making (virtual) friends and influencing people computational systems that establish rapport”
Wed Mar 11
7pm
Trayes Hall, Douglass College Center

**Review of HW, Practice Midterm**

Midterm Monday
- Here
- Covers Ch 1-3
- Open notes, book
- No electronic devices
**Test taking points**

Watch time
- I'll keep rough clock on chalkboard if needed

Read over all questions
- Start with ones you know how to do

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**1. Sizes**

Motion data tracks the position of markers as they move through space over time.

To track a person, you use 20 markers, and record their position 30 times per second.

Each marker has (x,y,z) position, each coordinate represented as a 4 byte number.

How many bits are needed for 10 minutes of this motion data?
Answer

8 bits/Byte ×
12 Bytes/Marker ×
20 Markers/Frame ×
30 Frames/Second ×
60 Seconds/Minute ×
10 Minutes =

34,560,000 bits

2. Truth tables

Write a logical expression (C = something that can include As, Bs, ands, ors, and nots) to match this truth table:

<table>
<thead>
<tr>
<th>A</th>
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<th>C</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>
2. Answer(s)

C = not A
C = (not A and not B) or (not A and B)

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3. Truth tables

Write a logical expression (C = something that can include As, Bs, ands, ors, and nots) to match this truth table:

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</tr>
</tbody>
</table>
4. Boolean formulas

Suppose $P = True$, $Q = False$, $R = True$, $S = False$.

Give the truth value (True or False) for each of these formulas:

1. $(Q \lor (not P \lor S)) \land (R \land S)$
2. $(not (R \land not (P \land Q)) \lor not S) \lor (Q \land not S)$
3. $(P \lor (Q \land R)) \land not (Q \lor (R \land S))$
Answer

Suppose P = True, Q = False, R = True, S = False.

(Q or (not P or S)) and (R and S)
(F or (not T or F)) and (T and F)
(F or (F or F)) and F
(F or F) and F
F and F
F

Answer

Suppose P = True, Q = False, R = True, S = False.

(not (R and not (P and Q)) or not S) or
(Q and not S)
(not (T and not (T and F)) or not F) or (F and not F)
(not (T and not F) or T) or (F and T)
(not (T and T) or T) or F
(not T or T) or F
(F or T) or F
T or F
T
Answer

Suppose P = True, Q = False, R = True, S = False.

(P or (Q and R)) and not (Q or (R and S))
(T or (F and T)) and not (F or (T and F))
(T or F) and not (F or F)
T and not F
T and T
T

5. Binary to decimal

Convert these 8-bit binary numbers to decimal
A. 1 1 1 0 0 1 1 0
B. 0 1 0 1 1 1 0 1
C. 0 1 1 0 0 0 1 1
D. 0 0 1 0 0 1 0 0
Answers

Convert these 8-bit binary numbers to decimal

A. 1 1 1 0 0 1 1 0 = 230 or -26
   128 + 64 + 32 + 4 + 2 = 230 (pos) or
   (sub 1: 11100101; flip: 00011010; 16+8+2=26)

B. 0 1 0 1 1 1 0 1 = 93
   64 + 16 + 8 + 4 + 1

C. 0 1 1 0 0 0 1 1 = 99
   64 + 32 + 2 + 1

D. 0 0 1 0 0 1 0 0 = 36
   32 + 4

6. Paint by numbers

First, convert the following decimals into 5-bit decimal numbers:
   (1) 30.  (2) 9.  (3) 14.  (4) 10.  (5) 25

Second, write the binary numbers on a 5x5 grid, one number per line, one bit per box.
Color the “1” boxes black and the others white.

(6) What is the picture?
7. New Year’s Eve

A NYE counter counts down to zero: 3, 2, 1, 0!
  • and then starts counting down for “next year”

Design a NYE counter circuit.
  • The input is two bits (AB) to hold the binary numbers from 0 to 3.
  • The output is two more bits (CD) for another binary number from 0 to 3.

(question continues on next page)
7. New Year’s Eve

Give the truth table describing AB and CD.

Write a formula for C as a function of A and B.

Write a formula for D as a function of A and B.

Answer

Give the truth table describing AB and CD.

<table>
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<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
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<td>0</td>
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<td>1</td>
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<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Answer

Write a formula for C as a function of A and B.

C = (not A and not B) or (A and B)

Write a formula for D as a function of A and B.

D = not B

Current HW

What is the result of the following binary additions:
1. 01011 + 10011
2. 1100110 + 0110101
3. 1111 + 101
Give your answers as binary numerals.
Current HW

What is the result of the following binary multiplications:
1. 1101 x 1001
2. 10100 x 1011
Give your answers as binary numerals.

Current HW

Use 8-bit binary numerals and the "twos complement" encoding of negative numbers that we studied in class to carry out the following subtraction problems:
1. 01100101 - 00001011
2. 00111011 - 00100111
Give your answers as binary numerals.
Current HW

Give a Boolean expression that describes the final value of E in terms of the initial values of A and B, for the following program of instructions:

```plaintext
acc = A.
acc = acc and B.
C = acc.
acc = not A.
D = acc.
acc = not B.
acc = acc and D.
acc = acc or C.
E = acc.
```

Current HW

Write a set of machine instructions in the language we studied in class, for the logic-accumulator CPU, to capture the following Boolean equation:

```
E = not (not A and not B)
```
Consider a state machine (like a counter or CPU), which updates the values of two variables. Suppose the state of the machine at one step is given by two bits A and B. Then the value of A at the next step will be given by the formula not A or B and the value of B at the next step will be given by the formula A and B.

1. Write out the truth table that describes this state machine, in four columns, giving in turn the values of A, B, not A or B (next A), and A and B (next B). (Again, please format your answer with a period or semicolon at the end of each line.)

2. Suppose the state machine starts in the state 11. When will the machine next be in state 11? Suppose the machine starts in state 00. When will the machine next be in state 00? What about state 01?

For both of the following scratch programs, say how many times the meow sound will be played and what the final value of x will be.
**Current HW**

For both of the following scratch programs, say how many times the meow sound will be played and what the final value of $x$ will be.
Consider a mouse pointer. One hundred times per second, it sends a position update as 2 bytes of $x$ information and 2 bytes of $y$ information, as well as a bit for each of its three buttons saying whether it is pressed. How many bits does the mouse send every minute?
Answer

\[(2 \times 8 + 2 \times 8 + 3)\text{ bits/message} \times\]
\[100\text{ messages/second} \times\]
\[60\text{ second/minute} =\]
\[210000\text{ bits/minute}\]

Practice Midterm - 2

Build the truth table for the formula
\[(A \text{ or not } B) \text{ and (not } A \text{ or } B)\]
Answer

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A or not B (^{(1)})</th>
<th>not A or B (^{(2)})</th>
<th>1 and 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
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<td>F</td>
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Practice Midterm - 3

Construct a Boolean equation describing \( E \) to an expression involving \( A, B, \) and \( C \), so to capture the following truth table:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>E</th>
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</thead>
<tbody>
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<td>F</td>
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Answer

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</table>

\[ E = \begin{cases} 
(\text{not } A \text{ and not } B \text{ and not } C) \\
\text{or} \\
(\text{not } A \text{ and } B \text{ and } C) \\
\text{or} \\
(A \text{ and not } B \text{ and } C) 
\end{cases} \]

Practice midterm - 4

Draw a circuit with and, or and not gates to compute an output \( E \) as a function of inputs \( A \) and \( B \) according to the following truth table:

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</table>
Answer

Observe: \( E = \text{not (A and B)} \)

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</table>

Answer

Also correct:
\( E = (\text{not A and not B}) \) or \( (\text{not A and B}) \) or \( (A \text{ and not B}) \)

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</table>
**To simplify**

(not A and not B) or (not A and B) or (A and not B)

Join terms:

(not A and not B) or (not A and B) = not A
(not A and not B) or (A and not B) = not B

So:

(not A and not B) or (not A and B) or (A and not B) =
(not A and not B) or (not A and B) or
(not A and not B) or (A and not B) =
not A or not B
De Morgan: not A or not B = not (A and B)

**Practice Midterm - 5**

We are building a state machine to display numbers in a specific sequence:

0, 3, 1, 2, 0, 3, 1, 2, . . .

Suppose the numbers are represented as two bit binary numerals: a higher order bit A and a lower order bit B. Write the truth table for the values of A and B at the next step as a function of their values at the current step.
(a) Express the decimal numeral 44 as a binary numeral.
(b) Express the binary numeral 1100111 as a decimal number.
**Answer**

(a) Express the decimal numeral 44 as a binary numeral.

$$101100 = 32 + 8 + 4$$

(b) Express the binary numeral 1100111 as a decimal number.

$$64 + 32 + 4 + 2 + 1 = 103$$

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**Practice Midterm - 7**

Carry out the following binary arithmetic.
(a) $1100111 + 101011011$
(b) $110010 \times 1110$
(c) $01110111 - 00011100$ (8 bit numbers)
Answers

Carry out the following binary arithmetic.

```
1100111
+ 101011011
111000010
```

Answers

Carry out the following binary arithmetic.

```
110010
× 1110
110010
110010
110010
110010
1010111100
```
Answer

(c) 01110111 - 00011100 (8 bit numbers)

Convert 00011100 to 11100011 plus 1 = 11100100

01110111
+
11100100
01011011

Practice Midterm - 8

Construct a program for our Boolean logic CPU, to express the formula

\[ E = (A \text{ or } B \text{ or } C) \text{ and not } D \]

Your answer will be a sequence of instructions of the form \( X = \text{acc}, \text{acc} = X, \text{acc} = \text{not} X, \text{and acc} = \text{acc op} X \), where \( X \) can be any variable and \( op \) can be \text{and or or}. 
Answer

acc = A
acc = acc or B
acc = acc or C
E = acc
acc = not D
acc = acc and E
E = acc

Practice Midterm - 9

How many times does the following scratch program play *meow*? How many times does it play *woof*?
**Answer: 6 meows, 6 woofs**

First repeat: count = 0
- 2 meows, count ← count + 2

Second repeat: count = 2
- 3 woofs, count ← count + 3

Third repeat: count = 5
- 2 meows, count ← count + 2

Fourth repeat: count = 7
- 3 woofs, count ← count + 3

Fifth repeat: count = 10
- 2 meows, count ← count + 2

Until condition now satisfied, stop
Practice Midterm - 10

Draw the picture created by the following scratch program. Angle 0 is up.

```
when play started
  clear
  pen down
  point in direction 0
  move 100 steps
  turn ↓ 120 degrees
  move 200 steps
  turn ↓ 120 degrees
  move 100 steps
  turn ↓ 120 degrees
  move 200 steps
  hide
  stop all
```
Answer